Solusi

Jawaban diberi tanda: $\square$

Untuk menghemat ruang, $90° - \theta$ ditulis θ_c; $180° - \theta$ ditulis θ_s

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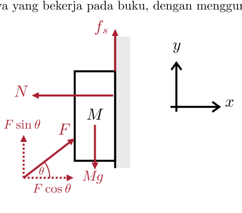
Pemanasan

1. **Menahan Buku (*).** Gambarkan gaya-gaya yang bekerja pada buku, dengan menggunakan sistem koordinat Kartesius:



Hukum I Newton:

$$\begin{aligned} \Sigma F_x &= 0 \\ F\cos\theta - N &= 0 \\ N &= F\cos\theta \cdots (1) \end{aligned} \qquad \begin{aligned} \Sigma F_y &= 0 \\ f_s + F\sin\theta - Mg &= 0 \\ f_s &= Mg - F\sin\theta \cdots (2) \end{aligned}$$

Berdasarkan definisi gaya gesek statis,

$$f_s \le \mu_s N$$

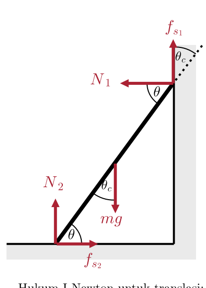
Substitusikan Pers. (1) dan (2):

$$\begin{aligned} Mg - F\sin\theta &\le \mu_s F\cos\theta \\ Mg &\le \mu_s F\cos\theta + F\sin\theta \\ Mg &\le (\mu_s\cos\theta + \sin\theta)F \\ \frac{Mg}{\mu_s\cos\theta+\sin\theta} &\le F \\ F &\ge \frac{Mg}{\mu_s\cos\theta+\sin\theta} \end{aligned} \qquad \square$$

Bagaimana cara mencari nilai θ supaya F minimum? Perhatikan bahwa M, g, dan μ_s nilainya selalu tetap sedangkan θ bisa diubah-ubah. Pecahan $\dfrac{Mg}{\mu_s\cos\theta+\sin\theta}$ akan bernilai minimum ketika penyebutnya maksimum. Nilai maksimum dari penyebutnya bisa dicari dengan menurunkannya terhadap θ dan menyamakannya dengan nol:

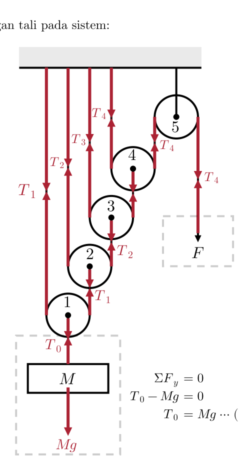
$$\begin{aligned} \frac{d}{d\theta}(\mu_s\cos\theta+\sin\theta) &= 0 \\ -\mu_s\sin\theta + \cos\theta &= 0 \\ \cos\theta &= \mu_s\sin\theta \\ \tan\theta &= \frac{1}{\mu_s} \\ \theta &= \arctan\left(\frac{1}{\mu_s}\right) \end{aligned} \qquad \square$$

2. **Tongkat Disandarkan pada Dinding (**).** Gambarkan gaya-gaya yang bekerja pada tongkat, dengan menggunakan sistem koordinat Kartesius:

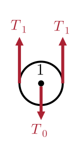


[illegible]

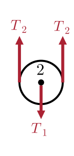
3. **Sistem Lima Katrol (*).** [illegible]



[illegible]

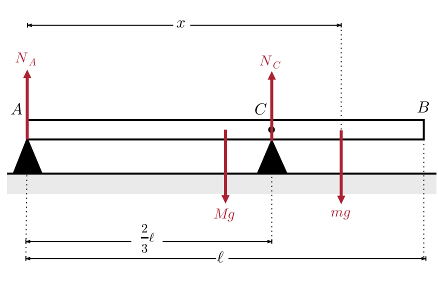


[illegible]



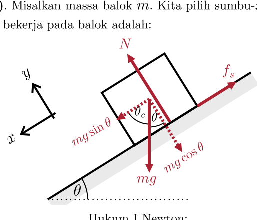
[illegible]

4. **Orang pada Jungkat-jungkit (**).** [illegible]



[illegible]

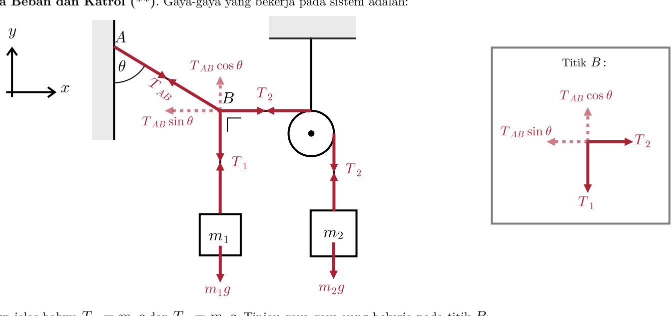
5. **Gesekan pada Bidang Miring (**).** [illegible]



[illegible]

[illegible]

1. **Dua Beban dan Katrol (**).** [illegible]



[illegible]

$$T_{AB} = \sqrt{m_1^2 + m_2^2 g}$$ $\square$

Nilai θ dapat dicari dengan mensubstitusikan T_{AB} ke Persamaan (1) ataupun (2):

$$\begin{aligned} \sqrt{m_1^2+m_2^2}\cancel{g}\sin\theta &= m_2\cancel{g} \\ \sin\theta &= \frac{m_2}{\sqrt{m_1^2+m_2^2}} \\ \theta &= \arcsin\left(\frac{m_2}{\sqrt{m_1^2+m_2^2}}\right) \quad \square \end{aligned} \qquad \begin{aligned} \sqrt{m_1^2+m_2^2}\cancel{g}\cos\theta &= m_1\cancel{g} \\ \cos\theta &= \frac{m_1}{\sqrt{m_1^2+m_2^2}} \\ \theta &= \arccos\left(\frac{m_1}{\sqrt{m_1^2+m_2^2}}\right) \quad \square \end{aligned}$$

Alternatif kedua:

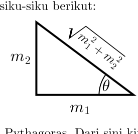
$$(1) \div (2):$$

$$\frac{T_{AB}\sin\theta}{T_{AB}\cos\theta} = \frac{m_2 g}{m_1 g}$$

$$\tan\theta = \frac{m_2}{m_1}$$

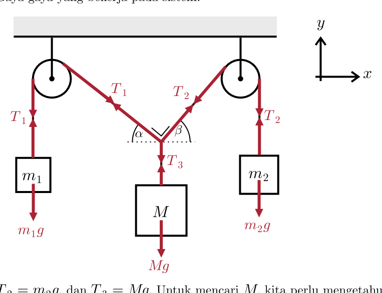
$$\theta = \arctan\left(\frac{m_2}{m_1}\right) \qquad \square$$

Secara grafis, ini ekuivalen dengan segitiga siku-siku berikut:

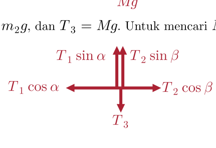


Sisi miringnya dapat dicari dengan teorema Pythagoras. Dari sini kita akan dapatkan nilai $\sin\theta$ dan $\cos\theta$ yang sama seperti pada alternatif pertama. Kita lalu bisa mencari T_{AB} dengan mensubstitusikannya ke Pers. (1) atau (2). Hasilnya akan sama. Ketiga nilai θ yang kita dapat sebenarnya sama saja. Sebagai contoh, pilih $m_1 = 1$ kg dan $m_2 = \sqrt{3}$ kg. Kita akan dapatkan $\theta = 60°$.

2. **Tiga Beban dan Katrol (*).** Gaya-gaya yang bekerja pada sistem:



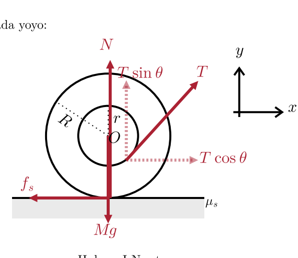
Cukup jelas bahwa $T_1 = m_1 g$, $T_2 = m_2 g$, dan $T_3 = Mg$. Untuk mencari M, kita perlu mengetahui nilai T_3:



Hukum I Newton:

$$\begin{aligned} \Sigma F_y &= 0 \\ T_1\sin\alpha + T_2\sin\beta - T_3 &= 0 \\ T_3 &= T_1\sin\alpha + T_2\sin\beta \\ Mg &= m_1 g\sin\alpha + m_2 g\sin\beta \\ M &= m_1\sin\alpha + m_2\sin\beta \quad \square \end{aligned}$$

3. **Yoyo (**).** Gaya-gaya yang bekerja pada yoyo:



Hukum I Newton:

$$\begin{aligned} \Sigma F_x &= 0 \\ T\cos\theta - f_s &= 0 \\ \cos\theta &= \frac{f_s}{T} \cdots (1) \end{aligned} \qquad \begin{aligned} \Sigma F_y &= 0 \\ N + T\sin\theta - Mg &= 0 \\ N \quad Mg &- T\sin\theta \cdots (2) \end{aligned} \qquad \begin{aligned} \Sigma\tau_O &= 0 \\ f_s R - Tr &= 0 \\ f_s &= \frac{r}{R}T \cdots (3) \end{aligned}$$

Substitusikan (3) ke (1):

$$\begin{aligned} \cos\theta &= \frac{\frac{r}{R}\cancel{T}}{\cancel{T}} = \frac{r}{R} \\ \theta &= \arccos\left(\frac{r}{R}\right) \quad \square \end{aligned}$$

Berdasarkan definisi gaya gesek statis:

$$f_s \le \mu_s N$$

$f_s = T\cos\theta$ (Pers. (1)) dan $N = Mg - T\sin\theta$ (Pers. (2)):

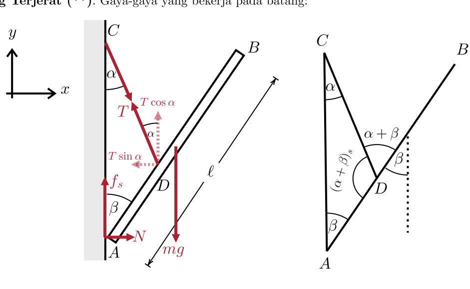
$$\begin{aligned} T\cos\theta &\le \mu_s(Mg - T\sin\theta) \\ T\cos\theta &\le \mu_s Mg - \mu_s T\sin\theta \end{aligned}$$

$$\begin{aligned} T\cos\theta + \mu_s T\sin\theta &\le \mu_s Mg \\ (\mu_s\sin\theta + \cos\theta)T &\le \mu_s Mg \\ T &\le \frac{\mu_s Mg}{\mu_s\sin\theta+\cos\theta} \end{aligned}$$

Nilai $\sin\theta$ tentunya dapat dicari karena kita sudah tahu nilai $\cos\theta$. $\sin\theta = \dfrac{\sqrt{R^2-1}}{R}$. Hasil akhirnya adalah:

$$T_{max} = \frac{\mu_s Mg}{\mu_s\cdot\frac{\sqrt{R^2-1}}{R} + \frac{r}{R}} = \frac{\mu_s MgR}{\mu_s\sqrt{R^2-1}+r} \qquad \square$$

4. **Batang yang Terjerat (**).** Gaya-gaya yang bekerja pada batang:



Hukum I Newton:

$$\begin{aligned} \Sigma F_x &= 0 \\ T\sin\alpha - N &= 0 \\ N &= T\sin\alpha \cdots (1) \end{aligned} \qquad \begin{aligned} \Sigma F_y &= 0 \\ T\cos\alpha + f_s - mg &= 0 \\ f_s &= mg - T\cos\alpha \cdots (2) \end{aligned}$$

$$\begin{aligned} \Sigma\tau_A &= 0 \\ T\sin(\alpha+\beta)\cdot\frac{\cancel{\ell}}{3} - mg\sin\beta\cdot\frac{\cancel{\ell}}{2} &= 0 \\ \frac{T\sin(\alpha+\beta)}{3} &= \frac{mg\sin\beta}{2} \\ T &= \frac{3mg\sin\beta}{2\sin(\alpha+\beta)} \cdots (3) \end{aligned}$$

Berdasarkan definisi gaya gesek statis:

$$f_s \le \mu_s N$$

Substitusikan persamaan (1) dan (2):

$$\begin{aligned} f_s &\le \mu_s N \\ mg - T\cos\alpha &\le \mu_s T\sin\alpha \\ \frac{mg - T\cos\alpha}{T\sin\alpha} &\le \mu_s \\ \mu_s &\ge \frac{mg}{T\sin\alpha} - \cot\alpha \end{aligned}$$

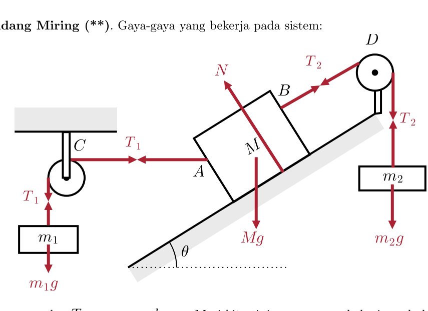
Substitusikan persamaan (3):

$$\mu_s \ge \frac{\cancel{mg}}{\frac{3\cancel{mg}\sin\beta}{2\sin(\alpha+\beta)}\sin\alpha} - \cot\alpha$$

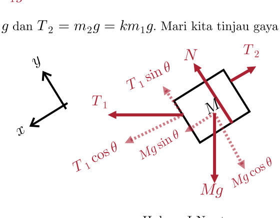
$$\begin{aligned} \mu_s &\ge \frac{2\sin(\alpha+\beta)}{3\sin\alpha\sin\beta} - \cot\alpha \\ \mu_s &\ge \frac{2}{3}\frac{\sin\alpha\cos\beta+\sin\beta\cos\alpha}{\sin\alpha\sin\beta} - \cot\alpha \\ \mu_s &\ge \frac{2}{3}\left(\frac{\cancel{\sin\alpha}\cos\beta}{\cancel{\sin\alpha}\sin\beta} + \frac{\cancel{\sin\beta}\cos\alpha}{\sin\alpha\cancel{\sin\beta}}\right) - \cot\alpha \\ \mu_s &\ge \frac{2}{3}(\cot\alpha+\cot\beta) - \cot\alpha \\ \mu_s &\ge -\frac{1}{3}\cot\alpha + \frac{2}{3}\cot\beta \\ \mu_s &\ge \frac{1}{3}|2\cot\beta - \cot\alpha| \end{aligned} \qquad \square$$

Kita menggunakan nilai mutlak untuk memastikan bahwa μ_s bernilai positif.

5. **Dua Katrol dan Bidang Miring (**).** Gaya-gaya yang bekerja pada sistem:



Cukup jelas bahwa $T_1 = m_1 g$ dan $T_2 = m_2 g = k m_1 g$. Mari kita tinjau gaya yang bekerja pada balok M:



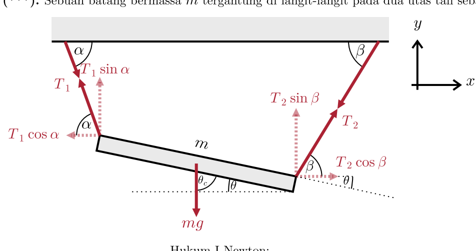
Hukum I Newton:

$$\begin{aligned} \Sigma F_x &= 0 \\ T_1\cos\theta + Mg\sin\theta - T_2 &= 0 \\ m_1\cancel{g}\cos\theta + M\cancel{g}\sin\theta - km_1\cancel{g} &= 0 \\ m_1\cos\theta - km_1 &= -M\sin\theta \\ (\cos\theta - k)m_1 &= -M\sin\theta \\ m_1 &= \frac{M\sin\theta}{k-\cos\theta} \end{aligned} \qquad \square$$

$$\begin{aligned} \Sigma F_y &= 0 \\ T_1\sin\theta + N - Mg\cos\theta &= 0 \\ N &= Mg\cos\theta - T_1\sin\theta \\ &= Mg\cos\theta - m_1 g\sin\theta \\ &= Mg\cos\theta - \frac{Mg\sin^2\theta}{k-\cos\theta} \\ &= \frac{Mg\cos\theta\cdot(k-\cos\theta) - Mg\sin^2\theta}{k-\cos\theta} \end{aligned}$$

$$\begin{aligned} &= \frac{Mgk\cos\theta - Mg\cos^2\theta - Mg\sin^2\theta}{k-\cos\theta} \\ &= \frac{Mgk\cos\theta - Mg\overbrace{(\sin^2\theta+\cos^2\theta)}^{1}}{k-\cos\theta} \\ N &= \frac{Mg(k\cos\theta - 1)}{k-\cos\theta} \end{aligned} \qquad \square$$

6. **Batang Tergantung (***).** Sebuah batang bermassa m tergantung di langit-langit pada dua utas tali sebagai berikut:



Hukum I Newton:

$$\begin{aligned} \Sigma F_x &= 0 \\ T_1\cos\alpha - T_2\cos\beta &= 0 \\ T_1 &= T_2\frac{\cos\beta}{\cos\alpha} \cdots (1) \end{aligned} \qquad \begin{aligned} \Sigma F_y &= 0 \\ T_1\sin\alpha + T_2\sin\beta - mg &= 0 \\ T_1\sin\alpha + T_2\sin\beta &= mg \cdots (2) \end{aligned}$$

Substitusikan Pers. (1) ke Pers. (2):

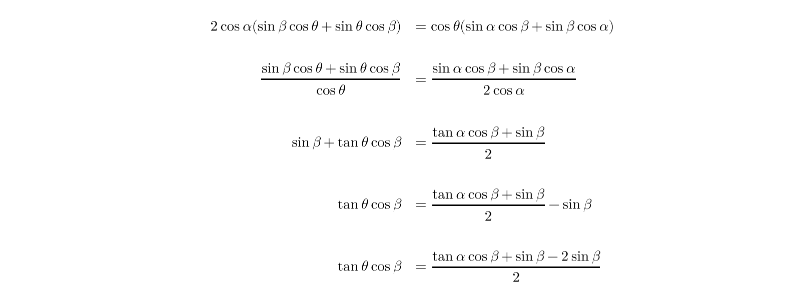
$$\begin{aligned} T_2\frac{\cos\beta}{\cos\alpha}\sin\alpha + T_2\sin\beta &= mg \\ \frac{T_2\cos\beta\sin\alpha + T_2\sin\beta\cos\alpha}{\cos\alpha} &= mg \\ T_2(\cos\beta\sin\alpha + \sin\beta\cos\alpha) &= mg\cos\alpha \\ T_2\sin(\alpha+\beta) &= mg\cos\alpha \\ T_2 &= \frac{mg\cos\alpha}{\sin(\alpha+\beta)} \cdots (3) \quad \square \end{aligned}$$

Substitusikan Pers. (3) ke Pers. (1):

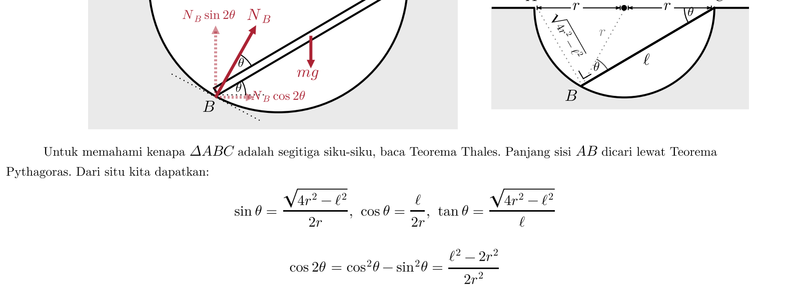
$$T_1 = \frac{mg\cos\beta}{\sin(\alpha+\beta)} \qquad \square$$

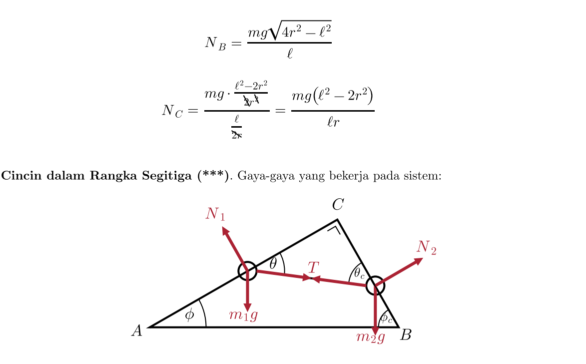
Melihat persamaan-persamaan di atas, kita tidak akan bisa menemukan θ dengannya. Kita perlu satu persamaan lagi. Karena batang setimbang, kita bisa menggunakan kesetimbangan torsi juga. Misalkan panjang batang adalah ℓ. Karena batang homogen, gaya berat bekerja pada tengah-tengah batang, yakni pada jarak $\ell/2$ dari ujung-ujungnya. Kita bisa tinjau torsi pada salah satu ujung batang, misalnya pada ujung tali T_1:

7. **Batang dalam Setengah Lingkaran (***)**. Gaya-gaya yang bekerja pada batang:

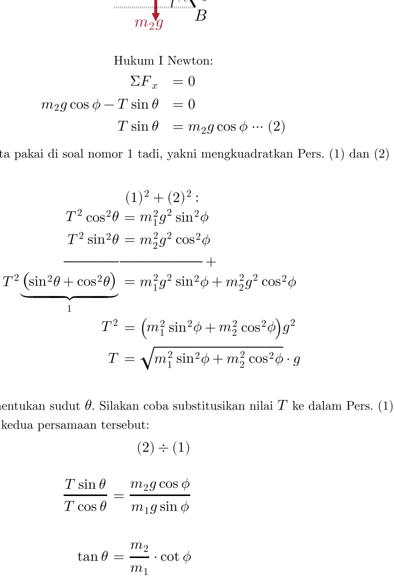


8. **Sepasang Cincin dalam Rangka Segitiga (***)**. Gaya-gaya yang bekerja pada sistem:





9. **Bola Terjepit (***)**. Gaya-gaya yang bekerja pada bola:



Hukum I Newton:

$$\Sigma F_x = 0 \qquad N_2 \sin\theta_2 - N_1 \sin\theta_1 = 0 \qquad N_1 \sin\theta_1 = N_2 \sin\theta_2 \cdots (1)$$

$$\Sigma F_y = 0 \qquad N_1 \cos\theta_1 - N_2 \cos\theta_2 - w = 0 \qquad N_1 \cos\theta_1 = N_2 \cos\theta_2 + w \cdots (2)$$

Bagi Pers. (1) dengan (2):

$$\frac{\cancel{N_1}\sin\theta_1}{\cancel{N_1}\cos\theta_1} = \frac{N_2 \sin\theta_2}{N_2\cos\theta_2 + w}$$

$$(N_2\cos\theta_2 + w)\sin\theta_1 = N_2 \sin\theta_2 \cos\theta_1$$

$$N_2 \cos\theta_2 \sin\theta_1 + w\sin\theta_1 = N_2 \sin\theta_2\cos\theta_1$$

$$w\sin\theta_1 = N_2\sin\theta_2\cos\theta_1 - N_2\cos\theta_2\sin\theta_1$$

$$w\sin\theta_1 = N_2(\sin\theta_2\cos\theta_1 - \cos\theta_2\sin\theta_1)$$

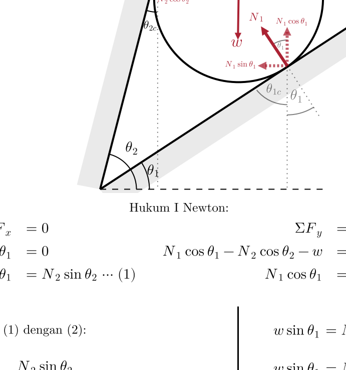
$$w\sin\theta_1 = N_2\sin(\theta_2 - \theta_1)$$

$$N_2 = \frac{w\sin\theta_1}{\sin(\theta_2-\theta_1)} \quad \square$$

Mensubstitusikannya ke Pers. (1) memberikan kita

$$N_1 = \frac{w\sin\theta_2}{\sin(\theta_2-\theta_1)} \quad \square$$

10. **Bola Terjepit 2.0 (***)**. Gaya-gaya yang bekerja pada bola:



Perhatikan bahwa

$$\theta_a + \theta_b + 90° = 180° \Leftrightarrow \theta_a + \theta_b = 90°$$

Hukum I Newton:

$$\Sigma F_x = 0 \qquad N_a \sin\theta_a - N_b\sin\theta_b = 0 \qquad N_a\sin\theta_a = N_b\sin\theta_b \cdots (1)$$

$$\Sigma F_y = 0 \qquad N_a\cos\theta_a + N_b\cos\theta_b - w = 0 \qquad N_a\cos\theta_a = -N_b\cos\theta_b + w \cdots (2)$$

Bagi Pers. (1) dengan (2):

$$\frac{\cancel{N_a}\sin\theta_a}{\cancel{N_a}\cos\theta_a} = \frac{N_b\sin\theta_b}{-N_b\cos\theta_b + w}$$

$$(w - N_b\cos\theta_b)\sin\theta_a = N_b\sin\theta_b\cos\theta_a$$

$$w\sin\theta_a = N_b\sin\theta_b\cos\theta_a + N_b\cos\theta_b\sin\theta_a$$

$$W\sin\theta_a = N_b(\sin\theta_b\cos\theta_a + \cos\theta_b\sin\theta_a)$$

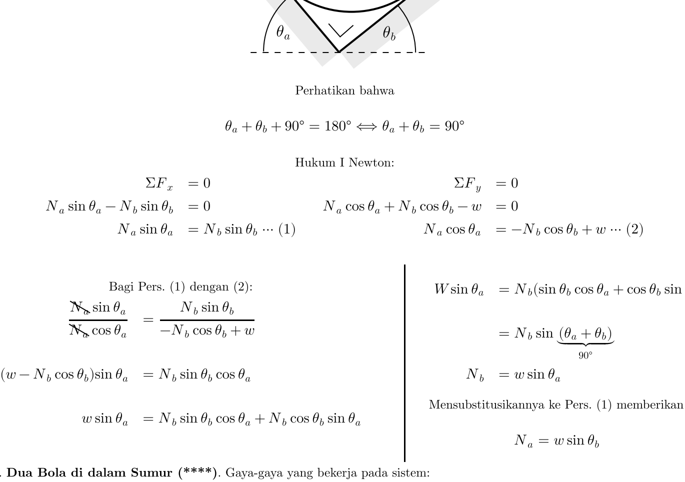
$$= N_b \sin\underbrace{(\theta_a+\theta_b)}_{90°}$$

$$N_b = w\sin\theta_a \quad \square$$

Mensubstitusikannya ke Pers. (1) memberikan kita

$$N_a = w\sin\theta_b \quad \square$$

11. **Dua Bola di dalam Sumur (****)**. Gaya-gaya yang bekerja pada sistem:



Dari gambar sebelah kanan, kita dapatkan

$$\sin\theta = \frac{\sqrt{2d(r_1+r_2) - d^2}}{r_1+r_2},\ \cos\theta = \frac{d-(r_1+r_2)}{r_1+r_2},\ \tan\theta = \frac{\sqrt{2d(r_1+r_2)-d^2}}{d-(r_1+r_2)}$$

Kita tinjau gaya pada tiap bola sendiri-sendiri:

Bola 1:

Hukum I Newton:

$$\Sigma F_x = 0 \qquad N_A - N_D\cos\theta = 0 \qquad N_A = N_D\cos\theta \cdots (1)$$

$$\Sigma F_y = 0 \qquad N_D\sin\theta - w_1 = 0 \qquad N_D = \frac{w_1}{\sin\theta} \cdots (2) \quad \square$$

Bola 2:

Hukum I Newton:

$$\Sigma F_x = 0 \qquad N_D\cos\theta - N_C = 0 \qquad N_C = N_D\cos\theta \cdots (3)$$

$$\Sigma F_y = 0 \qquad N_B - N_D\sin\theta - w_2 = 0 \qquad N_B = N_D\sin\theta + w_2 \cdots (4)$$

Substitusikan Pers. (2) ke tiga persamaan lainnya:

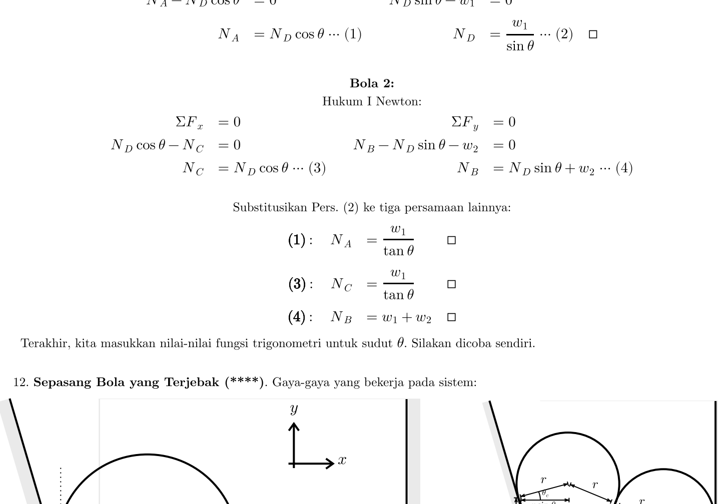
$$(1):\quad N_A = \frac{w_1}{\tan\theta} \quad \square$$

$$(3):\quad N_C = \frac{w_1}{\tan\theta} \quad \square$$

$$(4):\quad N_B = w_1 + w_2 \quad \square$$

Terakhir, kita masukkan nilai-nilai fungsi trigonometri untuk sudut θ. Silakan dicoba sendiri.

12. **Sepasang Bola yang Terjebak (****)**. Gaya-gaya yang bekerja pada sistem:



Dari gambar sebelah kanan, kita dapatkan

$$\sin\psi = \frac{\sqrt{4r^2-x^2}}{2r},\ \cos\psi = \frac{x}{2r},\ \tan\psi = \frac{\sqrt{4r^2-x^2}}{x}$$

Kita tinjau gaya pada tiap bola sendiri-sendiri:

Bola kiri:

Hukum I Newton:

$$\Sigma F_x = 0 \qquad N_A\sin\theta - N_D\cos\psi = 0 \qquad N_D\cos\psi = N_A\sin\theta \cdots (1)$$

$$\Sigma F_y = 0 \qquad N_D\sin\psi + N_A\cos\theta - w = 0 \qquad N_D\sin\psi = w - N_A\cos\theta \cdots (2) \quad \square$$

Bola kanan:

Hukum I Newton:

$$\Sigma F_x = 0 \qquad N_D\cos\psi - N_C = 0 \qquad N_C = N_D\cos\psi \cdots (3)$$

$$\Sigma F_y = 0 \qquad N_B - N_D\sin\psi - w = 0 \qquad N_B = N_D\sin\psi + w_2 \cdots (4)$$

Pers. (2) dibagi Pers. (1):

$$\frac{\cancel{N_D}\sin\psi}{\cancel{N_D}\cos\psi} = \frac{w - N_A\cos\theta}{N_A\sin\theta}$$

$$N_A\sin\theta\sin\psi = w\cos\psi - N_A\cos\theta\cos\psi$$

$$N_A\sin\theta\sin\psi + N_A\cos\theta\cos\psi = w\cos\psi$$

$$N_A(\cos\theta\cos\psi + \sin\theta\sin\psi) = w\cos\psi$$

$$N_A\cos(\theta-\psi) = w\cos\psi$$

$$N_A = \frac{w\cos\psi}{\cos(\theta-\psi)} \cdots (5) \quad \square$$

Substitusikan Pers. (5) ke (1):

$$N_D\cancel{\cos\psi} = \frac{w\cancel{\cos\psi}}{\cos(\theta-\psi)}\sin\theta$$

$$N_D = \frac{w\sin\theta}{\cos(\theta-\psi)} \cdots (6) \quad \square$$

Substitusikan Pers. (6) ke (3):

$$N_C = \frac{w\sin\theta\cos\psi}{\cos(\theta-\psi)} \quad \square$$

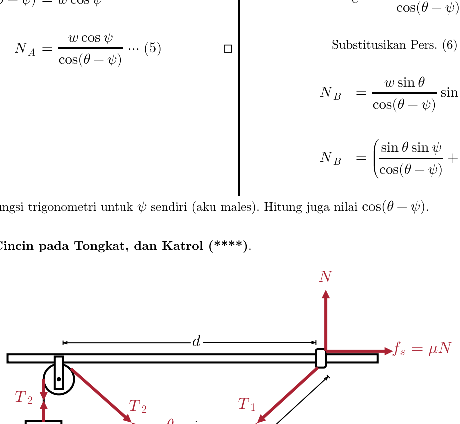
Substitusikan Pers. (6) ke (4):

$$N_B = \frac{w\sin\theta}{\cos(\theta-\psi)}\sin\psi + w$$

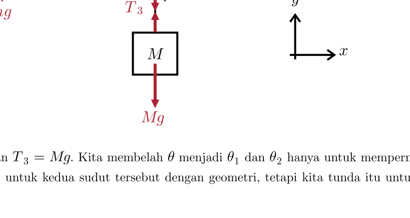
$$N_B = \left(\frac{\sin\theta\sin\psi}{\cos(\theta-\psi)} + 1\right)w \quad \square$$

Silakan coba substitusikan nilai fungsi trigonometri untuk ψ sendiri (aku males). Hitung juga nilai $\cos(\theta-\psi)$.

13. **Sistem Dua Beban, Cincin pada Tongkat, dan Katrol (****)**.



Cukup jelas bahwa $T_2 = mg$ dan $T_3 = Mg$. Kita membelah θ menjadi θ_1 dan θ_2 hanya untuk mempermudah analisis. Kita bisa mencari nilai fungsi trigonometri untuk kedua sudut tersebut dengan geometri, tetapi kita tunda itu untuk nanti dulu. Gaya-gaya yang bekerja pada cincin adalah:



Hukum I Newton:

$$\Sigma F_x = 0 \qquad \mu N - T_1\sin\theta_1 = 0 \qquad T_1\sin\theta_1 = \mu N \cdots (1)$$

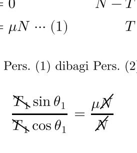
$$\Sigma F_y = 0 \qquad N - T_1\cos\theta_1 = 0 \qquad T_1\cos\theta_1 = N \cdots (2)$$

Pers. (1) dibagi Pers. (2):

$$\frac{\cancel{T_1}\sin\theta_1}{\cancel{T_1}\cos\theta_1} = \frac{\mu\cancel{N}}{\cancel{N}}$$

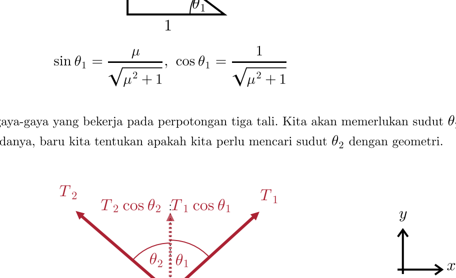
$$\tan\theta_1 = \mu \cdots (3)$$

Dari sini ternyata kita bisa dapatkan $\sin\theta_1$ dan $\cos\theta_1$ dengan segitiga siku-siku:



$$\sin\theta_1 = \frac{\mu}{\sqrt{\mu^2+1}},\ \cos\theta_1 = \frac{1}{\sqrt{\mu^2+1}}$$

Selanjutnya kita akan meninjau gaya-gaya yang bekerja pada perpotongan tiga tali. Kita akan memerlukan sudut θ_2. Akan tetapi, kita coba kerjakan dahulu apa adanya, baru kita tentukan apakah kita perlu mencari sudut θ_2 dengan geometri.



Hukum I Newton:

$$\Sigma F_x = 0 \qquad T_1\sin\theta_1 - T_2\sin\theta_2 = 0 \qquad T_1\sin\theta_1 = T_2\sin\theta_2 \qquad T_1\sin\theta_1 = mg\sin\theta_2 \qquad T_1 = mg\frac{\sin\theta_2}{\sin\theta_1} \cdots (4)$$

$$\Sigma F_y = 0 \qquad T_1\cos\theta_1 + T_2\cos\theta_2 - T_3 = 0 \qquad T_1\cos\theta_1 + T_2\cos\theta_2 = T_3 \qquad T_1\cos\theta_1 + mg\cos\theta_2 = Mg \cdots (5)$$

Substitusi Pers. (4) ke Pers. (5):

$$m\cancel{g}\frac{\sin\theta_2}{\sin\theta_1}\cos\theta_1 + m\cancel{g}\cos\theta_2 = M\cancel{g}$$

$$\frac{m\sin\theta_2\cos\theta_1 + m\cos\theta_2\sin\theta_1}{\sin\theta_1} = M$$

$$m(\sin\theta_2\cos\theta_1 + \cos\theta_2\sin\theta_1) = M\sin\theta_1$$

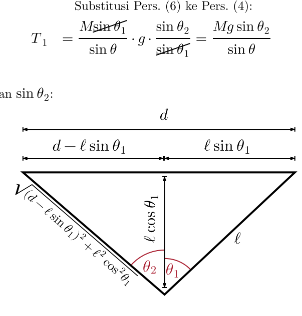
$$m\sin\underbrace{(\theta_1+\theta_2)}_{\theta} = M\sin\theta_1$$

$$m = \frac{M\sin\theta_1}{\sin\theta} = \frac{M\mu}{\sqrt{\mu^2+1}\cdot\sin\theta} \cdots (6) \quad \square$$

Substitusi Pers. (6) ke Pers. (4):

$$T_1 = \frac{M\cancel{\sin\theta_1}}{\sin\theta}\cdot g\cdot\frac{\sin\theta_2}{\cancel{\sin\theta_1}} = \frac{Mg\sin\theta_2}{\sin\theta} \quad \square$$

Dengan geometri, kita bisa temukan $\sin\theta_2$:



Di mana $\sqrt{(d-\ell\sin\theta_1)^2 + \ell^2\cos^2\theta_1}$ kita dapatkan dengan teorema Pythagoras. Dari sini bisa disimpulkan:

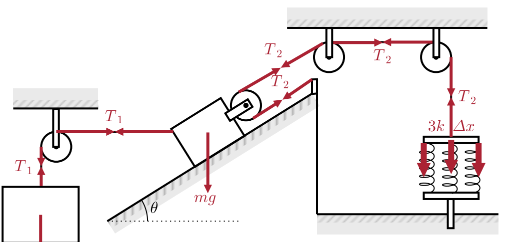
$$\sin\theta_2 = \frac{d-\ell\sin\theta_1}{\left[(d-\ell\sin\theta_1)^2 + \ell^2\cos^2\theta_1\right]^{1/2}} = \frac{d - \frac{\mu\ell}{\sqrt{\mu^2+1}}}{\left[\left(d-\frac{\mu\ell}{\sqrt{\mu^2+1}}\right)^2 + \left(\frac{\ell}{\sqrt{\mu^2+1}}\right)^2\right]^{1/2}}$$

$$\cos\theta_2 = \frac{\ell\cos\theta_1}{\left[(d-\ell\sin\theta_1)^2 + \ell^2\cos^2\theta_1\right]^{1/2}} = \frac{\frac{\ell}{\sqrt{\mu^2+1}}}{\left[\left(d-\frac{\mu\ell}{\sqrt{\mu^2+1}}\right)^2 + \left(\frac{\ell}{\sqrt{\mu^2+1}}\right)^2\right]^{1/2}}$$

Silakan coba disubstitusikan sendiri ke T_1. Nilai θ saat kondisi tersebut terjadi adalah

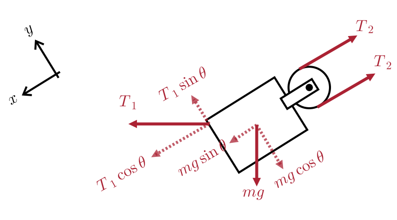
$$\begin{aligned}
\sin\theta &= \sin(\theta_1+\theta_2) \\
&= \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2 \\
&= \frac{\mu}{\left[\mu^2+1\right]^{1/2}}\cdot\frac{\frac{\ell}{\sqrt{\mu^2+1}}}{\left[\left(d-\frac{\mu\ell}{\sqrt{\mu^2+1}}\right)^2+\left(\frac{\ell}{\sqrt{\mu^2+1}}\right)^2\right]^{1/2}} + \frac{1}{\left[\mu^2+1\right]^{1/2}}\cdot\frac{d-\frac{\mu\ell}{\sqrt{\mu^2+1}}}{\left[\left(d-\frac{\mu\ell}{\sqrt{\mu^2+1}}\right)^2+\left(\frac{\ell}{\sqrt{\mu^2+1}}\right)^2\right]^{1/2}} \\
&= \frac{\cancel{\frac{\mu\ell}{\sqrt{\mu^2+1}}} + d - \cancel{\frac{\mu\ell}{\sqrt{\mu^2+1}}}}{\left[(\mu^2+1)\left(\left(d-\frac{\mu\ell}{\sqrt{\mu^2+1}}\right)^2+\left(\frac{\ell}{\sqrt{\mu^2+1}}\right)^2\right)\right]^{1/2}} = \frac{d}{\sqrt{\left(\sqrt{\mu^2+1}\right)^2\left(\left(d-\frac{\mu\ell}{\sqrt{\mu^2+1}}\right)^2+\left(\frac{\ell}{\sqrt{\mu^2+1}}\right)^2\right)}} \\
&= \frac{d}{\sqrt{\left(\sqrt{\mu^2+1}\cdot d - \mu\ell\right)^2 + \ell^2}} \quad\Leftrightarrow\quad \theta = \sin^{-1}\left(\frac{d}{\sqrt{\left(\sqrt{\mu^2+1}\cdot d-\mu\ell\right)^2+\ell^2}}\right) \quad \square
\end{aligned}$$

14. **Sistem Beban dan Pegas (***)**. Gaya-gaya yang bekerja pada sistem:



$3mg$

Cukup jelas bahwa $T_1 = 3mg$ dan $T_2 = 3k\Delta x$. Selanjutnya, kita analisis gaya-gaya yang bekerja pada balok m di bidang miring:



Hukum I Newton:

$$\begin{aligned} \Sigma F_x &= 0 \\ T_1 \cos\theta + mg\sin\theta - 2T_2 &= 0 \\ 3mg\cos\theta + mg\sin\theta - 6k\Delta x &= 0 \\ 6k\Delta x &= 3mg\cos\theta + mg\sin\theta \\ k &= \frac{3mg\cos\theta + mg\sin\theta}{6\Delta x} \quad \square \end{aligned}$$

15. **Tumpukan Balok pada Bidang Miring (**)**. Resultan gaya pada balok A maupun B pasti sama dengan nol. Balok B tertahan oleh tali, sedangkan balok A bergerak dengan kecepatan konstan (tanpa percepatan). Yuk, langsung kita tinjau gaya pada tiap balok:

Balok B:

$$\begin{aligned} \Sigma F_x &= 0 \\ f_2 + W\sin\theta - T &= 0 \\ T &= \mu_k N_2 + W\sin\theta \cdots (1) \end{aligned}$$

$$\begin{aligned} \Sigma F_y &= 0 \\ N_2 - W\cos\theta &= 0 \\ N_2 &= W\cos\theta \cdots (2) \end{aligned}$$

Balok A:

$$\begin{aligned} \Sigma F_x &= 0 \\ W\sin\theta - f_1 - f_2 &= 0 \\ \mu_k N_1 + \mu_k N_2 &= W\sin\theta \\ \mu_k (N_1 + N_2) &= W\sin\theta \cdots (3) \end{aligned}$$

$$\begin{aligned} \Sigma F_y &= 0 \\ N_1 - N_2 - W\cos\theta &= 0 \\ N_1 &= N_2 + W\cos\theta \cdots (4) \end{aligned}$$

Substitusikan (2) ke (4):

$$N_1 = 2W\cos\theta \cdots (5)$$

Substitusikan (2) dan (5) ke (3):

$$3\mu_k W\cos\theta = W\sin\theta$$

$$\mu_k = \frac{\tan\theta}{3} = \frac{\frac{3}{4}}{3} = \frac{1}{4} \quad \square$$

Substitusikan (2) dan μ_k ke (1):

$$\begin{aligned} T &= \frac{W\cos\theta}{4} + W\sin\theta \\ &= \frac{\frac{4}{5}W}{4} + \frac{3}{5}W \\ &= \frac{4W}{5} \quad \square \end{aligned}$$